Facultatea de Automatica si Calculatoare
Sectia: Calculatoare si Tehnologia Informatiei

Algebra si Geometrie

Timp alocat: 55 min | Semestrul I | Nota maxima: 4

Test seminar

1. (a) Aratati ca transformarea $T_{\boldsymbol{a}} : \mathbb{R}^3 \to \mathbb{R}^3$ definita prin

$$T_{\boldsymbol{a}}(\boldsymbol{v}) = \boldsymbol{a} \times \boldsymbol{v}, \qquad \forall\, \boldsymbol{v} \in \mathbb{R}^3,$$

unde $\boldsymbol{a} = (1,1,1) \in \mathbb{R}^3$, este o aplicatie liniara si aflati matricea asociata. Studiati injectivitatea si surjectivitatea acestei aplicatii.

(1 punct)

(b) Aratati ca are loc identitatea Grassmann

$$(\mathbf{a} \times \mathbf{b}) \times \mathbf{c} = \mathbf{a} \bullet \mathbf{c} \cdot \mathbf{b} - \mathbf{b} \bullet \mathbf{c} \cdot \mathbf{a}, \qquad \mathbf{a}, \mathbf{b}, \mathbf{c} \in \mathbb{R}^3$$

si argumentati apoi ca $(\mathbf{c} \times \mathbf{a}) \times \mathbf{b} \in \operatorname{span}\{\mathbf{a}, \mathbf{c}\}$.

(1 punct)

(c) Aflati o solutie aproximativa $\boldsymbol{x}^*$ prin metoda celor mai mici patrate pentru sistemul

$$\begin{cases} x + y = 0 \\ -x - y = 1 \end{cases}$$

(1 punct)

(d) Descompuneti matricea data sub forma $\boldsymbol{A} = \boldsymbol{Q} \cdot \boldsymbol{D} \cdot \boldsymbol{Q}^t$, unde $\boldsymbol{Q}$ este o matrice ortogonala si $\boldsymbol{D}$ o matrice diagonala

$$\boldsymbol{A} = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$$

(1 punct)